

Assignment #2

Due Date: Thursday, June 3rd, 2010 by 12 noon
(in Scurfield Hall, OPMA 403 drop-off box #20)

Instructor: Janice B. Eliasson

<i>Question</i>	<i>Mark</i>	<i>Maximum</i>
1		20
2		15
3		15
4		15
5		15
6		20
Total		100
Group Number	7	

Last Name (in alphabetical order)	First Name
Hope	Gillian
Khan	Omar
Nguyen	Nolan
Singleton	James

OPMA403 Spring 2010

Assignment #2 Question 1 (20 marks)

\bar{X} double bar (from question) = 100.10

R-bar (from question) = 5.60

$A_2 = 0.58$

$D_3 = 0.00$

$D_4 = 2.11$

Sample #	Observation					Mean	Range
	#1	#2	#3	#4	#5		
1	100.8	98.5	98.2	96.2	99.1	98.56	4.60
2	104.3	102.4	106.8	103.6	96.8	102.78	10.00
3	99.2	101.2	99.5	99.4	103.4	100.54	4.20
4	101.4	100.2	96.7	99.3	97.1	98.94	4.70
5	98	102.9	99.8	100.5	97.8	99.80	5.10
6	102.5	96.2	101.8	101.2	99.5	100.24	6.30
7	94.3	96.9	101.3	99.9	94.1	97.30	7.20
8	99.5	92.8	100.2	97.3	96.4	97.24	7.40
9	102.6	99.7	101	102.1	103.4	101.76	3.70
10	98.3	100.7	96.2	101.6	100.9	99.54	5.40

X-Chart Control Limits:

$$UCL_x = \bar{X} + A_2 R$$

$$UCL_x = 100.10 + 0.58 * 5.6$$

$$UCL_x = 103.35$$

$$LCL_x = \bar{X} - A_2 R$$

$$LCL_x = 100.10 - 0.58 * 5.6$$

$$LCL_x = 96.85$$

R-Chart Control Limits:

$$UCL_R = D_4 * R\text{-bar}$$

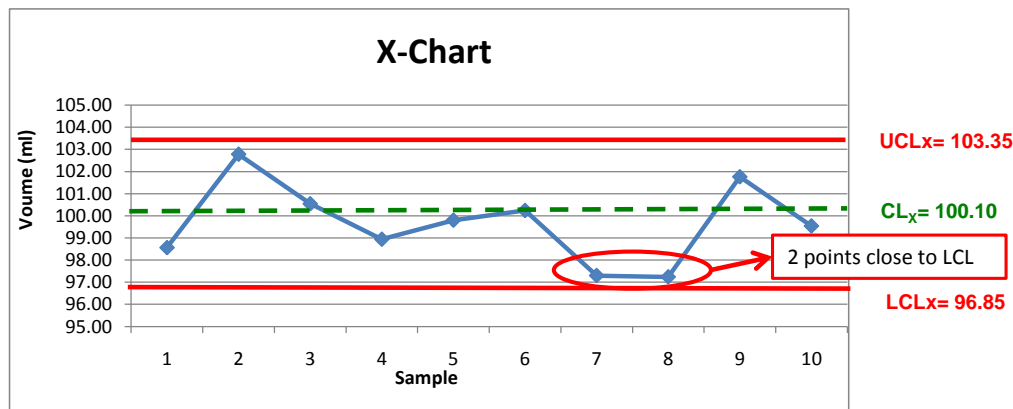
$$UCL_R = 2.11 * 5.6$$

$$UCL_R = 11.82$$

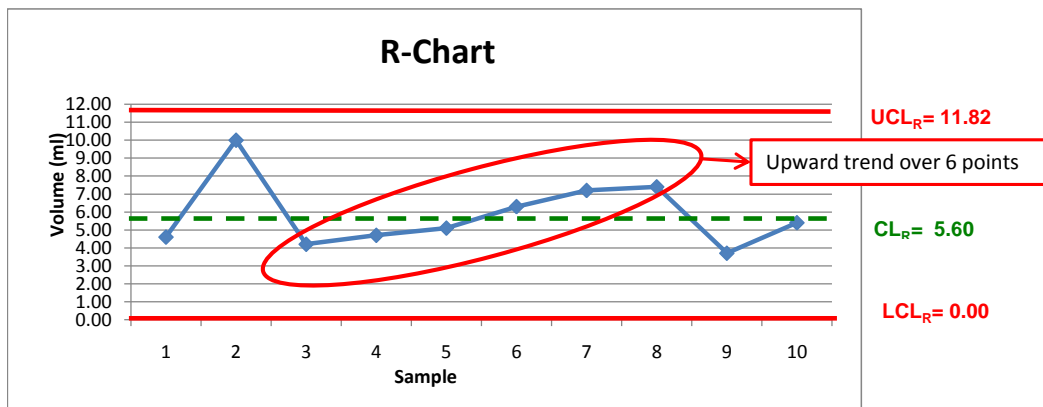
$$LCL_R = D_3 * R\text{-bar}$$

$$LCL_R = 0.00 * 5.6$$

$$LCL_R = 0.00$$



Concern about two consecutive points very close to LCL (samples 7,8). Although the X-chart is in control as all points are between the control limits, the two points close to the LCL require investigation.



Concern about increasing trend from sample 3 to sample 8. The R-chart is in control since all points are between control limits, but the trend requires investigation.

OPMA403 **Spring 2010**
Assignment #2 **Question 2**

Q.2 X&R Charts - OOC (15 marks)

$n = 11$ $m = 25$
 $A_2 = 0.29$ $D_3 = 0.26$ $D_4 = 1.74$

Subgroup #	X	R	Subgroup #	X	R	Subgroup #	X	R
1	12.33	0.31	10	12.37	0.40	19	12.45	0.38
2	12.38	0.57	11	12.41	0.55	20	12.46	0.44
3	12.37	0.37	12	12.37	0.37	21	12.33	0.52
4	12.33	0.43	13	12.16	0.51	22	12.40	0.44
5	12.97	0.93	14	12.44	0.38	23	12.42	0.48
6	12.32	0.32	15	12.36	0.34	24	12.53	0.84
7	12.45	0.35	16	12.39	0.35	25	12.36	0.38
8	12.44	0.31	17	12.44	0.31			
9	12.41	0.35	18	12.38	0.38	Total	310.27	11.01

$$\bar{\bar{X}} = \frac{\sum_{j=1}^m \bar{X}_j}{m} = 12.41 \text{ mm}$$

$$\bar{R} = \frac{\sum_{j=1}^m R_j}{m} = 0.44 \text{ mm}$$

$$UCL_X: \bar{\bar{X}} + A_2 \bar{R} = 12.54 \text{ mm} \quad UCL_R: D_4 \bar{R} = 0.77 \text{ mm}$$

$$LCL_X: \bar{\bar{X}} - A_2 \bar{R} = 12.28 \text{ mm} \quad LCL_R: D_3 \bar{R} = 0.11 \text{ mm}$$

OOB points: **5, 13, and 24** (highlighted above; bold points are OOB)

Revised

$$\bar{\bar{X}} = \frac{\sum_{j=1}^m \bar{X}_j}{m} = 12.39 \text{ mm}$$

$$\bar{R} = \frac{\sum_{j=1}^m R_j}{m} = 0.40 \text{ mm}$$

$$UCL_X: \bar{\bar{X}} + A_2 \bar{R} = 12.51 \text{ mm} \quad UCL_R: D_4 \bar{R} = 0.69 \text{ mm}$$

$$LCL_X: \bar{\bar{X}} - A_2 \bar{R} = 12.28 \text{ mm} \quad LCL_R: D_3 \bar{R} = 0.10 \text{ mm}$$

The process can now be monitored for control because all of the remaining points fall within the control limits.

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Assignment #2 Question 3 (15 marks)



Regular Shift Sample	# drills that are defective	2nd Shift Sample	# drills that are defective	Prob. (of 250)
1	4	1	5	2.00%
2	6	2	4	1.60%
3	6	3	8	3.20%
4	8	4	4	1.60%
5	6	5	12	4.80%
6	8	6	1	0.40%
7	6	7	8	3.20%
8	6	8	9	3.60%
9	5	9	7	2.80%
10	6	10	5	2.00%
11	5	11	2	0.80%
12	8	12	4	1.60%
13	5	13	3	1.20%
14	4	14	3	1.20%
15	5	15	7	2.80%
16	5	16	4	1.60%
17	6	17	10	4.00%
18	9	18	5	2.00%
19	6	19	7	2.80%
20	4	20	5	2.00%
21	7	21	9	3.60%

p-value: $\frac{\# \text{ defective}}{nm} = 0.0238$

Std Dev: $\left(\frac{p(1-p)}{n} \right)^{1/2} = 0.0096$

Control Limits

$$LCLp = p - 3S_p$$

$$LCLp = \bar{p} - 3 * \bar{p}_{std_dev}$$

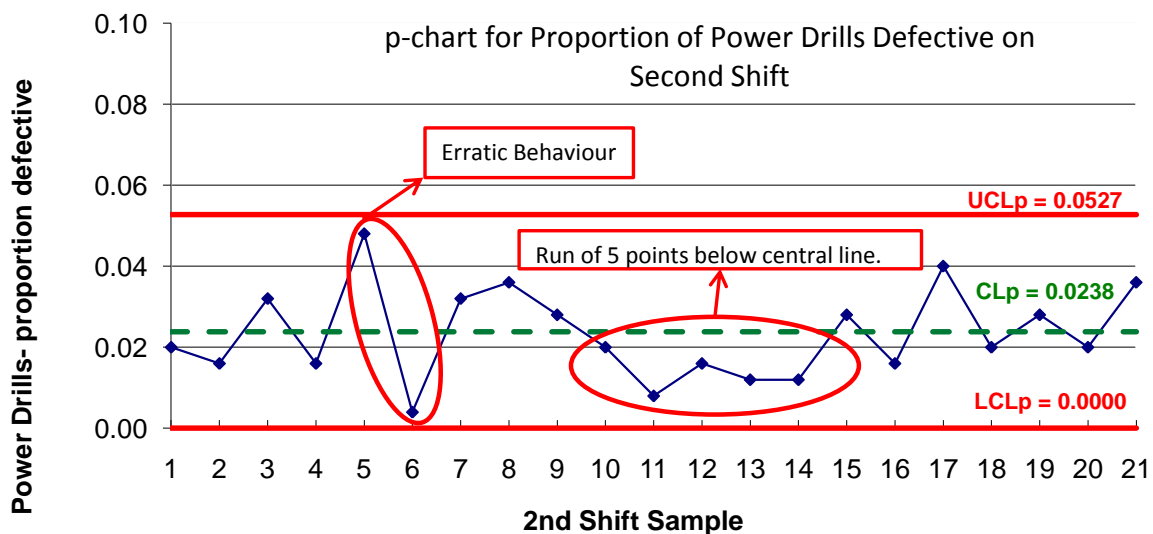
$$LCLp = 0.00$$

(Since LCL cannot be negative)

$$UCLp = p + 3S_p$$

$$UCLp = \bar{p} + (3) * \bar{p}_{std_dev}$$

$$UCLp = 0.0527$$



Concern about erratic behaviour between samples 5 and 6 (two consecutive points close to the control limits) as well as 5 consecutive points below the CL (samples 10-14). The process is in control, since no points are outside of the control limits, but the company should investigate in order to find assignable cause.

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Q. 4 C-charts (15 marks)

Serial Number	Count of Defects	Serial Number	Count of Defects
301	4	315	2
302	3	316	3
303	10	317	2
304	4	318	11
305	2	319	3
306	3	320	2
307	4	321	4
308	1	322	3
309	3	323	5
310	14	324	3
311	2	325	9
312	2	326	3
313	4		
314	5	Total	111

$$m = 26 \quad \Sigma c = 111$$

Establishing Limits

$$\bar{c} = \frac{\Sigma c}{m} = \frac{111}{26} = 4.3 \quad UCL_c: \bar{c} + 3\sqrt{\bar{c}} = 10.5$$

$$LCL_c: \bar{c} - 3\sqrt{\bar{c}} = 0.0 \quad (\text{since LCL cannot be negative})$$

OOc are 310, 318 (highlighted in RED)

Revised Limits 1

$$\bar{c} = \frac{\Sigma c}{m} = \frac{111 - 14 - 11}{26 - 2} = 3.6 \quad UCL_c: \bar{c} + 3\sqrt{\bar{c}} = 9.3$$

$$LCL_c: \bar{c} - 3\sqrt{\bar{c}} = 0.0 \quad (\text{since LCL cannot be negative})$$

OOc is 303 (highlighted in ORANGE)

Revised Limits 2

$$\bar{c} = \frac{\Sigma c}{m} = \frac{86 - 10}{24 - 1} = 3.3 \quad UCL_c: \bar{c} + 3\sqrt{\bar{c}} = 8.8$$

$$LCL_c: \bar{c} - 3\sqrt{\bar{c}} = 0.0 \quad (\text{since LCL cannot be negative})$$

OOc is 325 (highlighted in PURPLE)

Revised Limits 3

$$\bar{c} = \frac{\Sigma c}{m} = \frac{76 - 9}{23 - 1} = 3.0 \quad UCL_c: \bar{c} + 3\sqrt{\bar{c}} = 8.3$$

$$LCL_c: \bar{c} - 3\sqrt{\bar{c}} = 0.0 \quad (\text{since LCL cannot be negative})$$

The process can now be monitored for control because all of the remaining points fall within the control limits.

Q. 5 U-charts (15 marks)

Sample	Defects	Sample	Defects
1	5	10	1
2	24	11	8
3	2	12	7
4	5	13	6
5	8	14	2
6	8	15	6
7	11	16	8
8	8	17	5
9	5	Total	119

$$n = 12 \quad m = 17 \quad \sum c = 119$$

$$\bar{u} = \frac{\sum c}{nm} = \frac{119}{12(17)} = 0.58$$

$$UCL_U: \quad \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} \quad 1.24$$

$$LCL_U: \quad \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} \quad 0.00 \text{ (Since LCL cannot be negative)}$$

$$MAX = UCL * n = 14.9$$

$$MIN = LCL * n = 0.0$$

OOB sample is #2

$$\bar{u} = \frac{\sum c}{nm} = \frac{119 - 24}{12(16)} = 0.49$$

$$UCL_U: \quad \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} \quad 1.10$$

$$LCL_U: \quad \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} \quad 0.00 \text{ (Since LCL cannot be negative)}$$

$$MAX = UCL * n = 13.2$$

$$MIN = LCL * n = 0.0$$

The process can now be monitored for control because all of the remaining points fall within the control limits.

6.

$$a. \quad C_p = \frac{USL - LSL}{6\sigma} = \frac{31.2 - 28.7}{6(0.50)} = \frac{2.5}{3} = 0.83$$

The calculated value for $C_p = 0.83$ is below both the industry standard ($C_{p \text{ ind}} = 1.33$) and the minimum value required for the process to be considered capable ($C_{p \text{ min}} = 1.00$). This indicates that the manufacturer is scrapping/wasting parts and spending money on rework. As a result, the process is not considered capable.

$$b. \quad Z(USL) = \frac{USL - \bar{X}}{\sigma} = \frac{31.2 - 29.9}{0.50} = 2.6$$

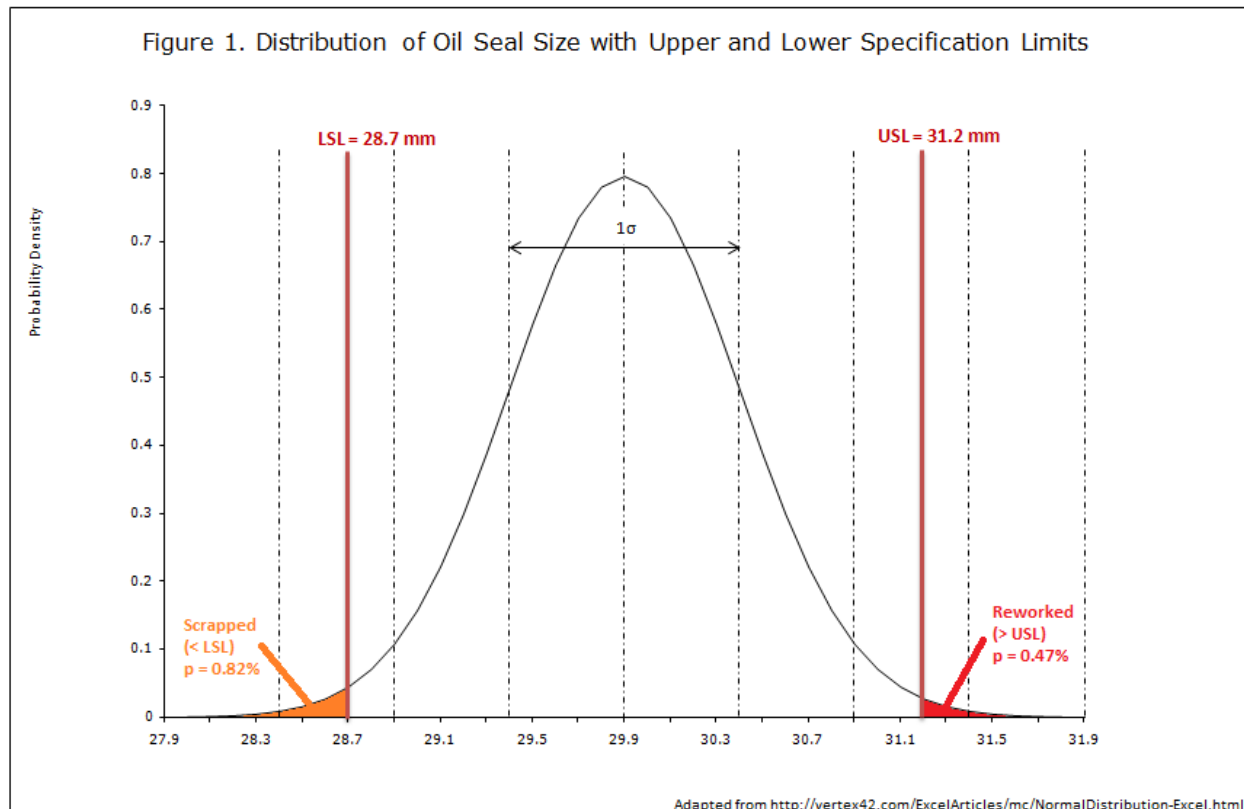
$$Z(LSL) = \frac{\bar{X} - LSL}{\sigma} = \frac{29.9 - 28.7}{0.50} = 2.4$$

$$C_{pk} = \frac{Z(\min)}{3} = \frac{2.4}{3} = 0.80$$

Since $C_p \neq C_{pk}$, the process cannot be considered to be centered.

- c. Since the oil seals are reworked if they are above the USL (ie. when they are too large) and are scrapped if they are below the LSL (ie. when they are too small), this operation is a **piece** operation.

d.



- e. Percentage reworked (ie. percentage above USL):

$$Z(USL) = \frac{USL - \mu}{\sigma} = \frac{31.2 - 29.9}{0.50} = 2.6$$

$$p(Z \leq 2.6) = 0.9953$$

$$p(Z > 2.6) = 1 - p(Z \leq 2.6) = 1 - 0.9953 = 0.0047 = 0.47\%$$

Therefore, 0.47% of all produced oil seals are reworked.

Percentage scrapped (ie. percentage below LSL):

$$Z(LSL) = \frac{LSL - \mu}{\sigma} = \frac{28.7 - 29.9}{0.50} = -2.4$$

$$p(Z \leq -2.4) = p(Z > 2.4)$$

$$p(Z \leq 2.4) = 0.9918$$

$$p(Z > 2.4) = p(Z < -2.4) = 1 - p(Z \leq 2.4) = 1 - 0.9918 = 0.0082 = 0.82\%$$

Therefore, 0.82% of all produced oil seals are scrapped.